Flight Dynamics & Control
Flight Control

Harry G. Kwatny
Department of Mechanical Engineering & Mechanics
Drexel University

Outline

- Types of Flight Control Systems
- Classical & Modern Design Methods
  - Classical SISO – Root locus, frequency domain (bode plots, sensitivity function, Nyquist analysis, gain & phase margins)
  - Modern SISO or MIMO – pole placement, optimal control (H_2, H_∞), separation principle
- Controllability/Observability
- Normal Forms for LTI Systems
- Pole Placement
  - 747 Altitude Hold
- Separation Principle
  - F-16 Stability Augmention System

Control Systems

- flight path regulation:
  - flight path angle, γ, via elevator, δ_e,
  - velocity regulation, V via thrust, T,
  - altitude hold, h via elevator, δ_e,
- autopilots: regulation of attitude using independent single loops
  - pitch, θ via elevator, δ_e,
  - roll, ϕ via aileron, δ_a,
  - yaw, ψ via rudder, δ_r,
- stability augmentation systems: feedback of angular rates
  - pitch, θ, via elevator, δ_e,
  - roll, ϕ, via aileron, δ_a,
  - yaw, ψ, via rudder, δ_r,

Drexel UNIVERSITY

Controllability & Observability

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Controllability: The system is (completely) controllable if there exists a control input $u(t)$ defined on a finite time interval $[0, T]$ that steers the system from any initial state x_0 to any final state x_1.

Observability: The system is (completely) observable if the intial state x_0 can be determined from knowledge of the input $u(t)$ and the measurement of the output $y(t)$ over a finite time interval $[0, T]$.

Drexel UNIVERSITY

Controllability / Observability Tests

Controllability Matrix: $\mathscr{C} = \begin{bmatrix} B & AB & \cdots & A^{n-1} \end{bmatrix}$

Observability Matrix: $\mathscr{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$

Controllable $\Leftrightarrow$ rank $\mathscr{C} = n$ (for SI $\det \mathscr{C} \neq 0$)

Observable $\Leftrightarrow$ rank $\mathscr{O} = n$ (for SO $\det \mathscr{O} \neq 0$)

Special Forms

Consider a SISO controllabile & observable system

$$\mathcal{C} = \begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix}, \det \mathcal{C} \neq 0 \quad \mathcal{O} = \begin{bmatrix} c \\ cA \\ \vdots \\ cA^{n-1} \end{bmatrix}, \det \mathcal{O} \neq 0$$

$$\mathcal{C}^{-1} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} \qquad \mathcal{O}^{-1} = \begin{bmatrix} p_1 & \cdots & p_n \end{bmatrix}$$

We will consider four state transformations defined by

$$T_1 = \mathcal{C}, T_2 = \mathcal{O}, T_3 = \begin{bmatrix} q_n \\ q_n A \\ \vdots \\ q_n A^{n-1} \end{bmatrix}, T_4 = \begin{bmatrix} p_n & Ap_n & \cdots & A^{n-1}p_n \end{bmatrix}$$

Controllability Form for SISO Systems

$$\dot{x} = Ax + bu,\ y = cx$$

$$\mathscr{C} = \begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix},\ \det \mathscr{C} \neq 0$$

$$T = \begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix} \qquad T = \begin{bmatrix} A^{n-1}b & \cdots & Ab & b \end{bmatrix}$$

$$\Downarrow \qquad \Downarrow$$

$$\dot{z} = \begin{bmatrix} 0 & & 0 & -a_0 \\ 1 & \ddots & & -a_1 \\ & \ddots & 0 & \vdots \\ 0 & & 1 & -a_{n-1} \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u \qquad \dot{z} = \begin{bmatrix} -a_{n-1} & 1 & & 0 \\ \vdots & 0 & \ddots & \\ -a_1 & & \ddots & 1 \\ -a_0 & 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = cTz$$

Drexel UNIVERSITY

Controllability Form – the transformation

$$\dot{z} = \left(T^{-1}AT\right)z + \left(T^{-1}b\right)u$$

$$T^{-1}T = I \Rightarrow \begin{bmatrix} T^{-1}b & T^{-1}Ab & \cdots & T^{-1}A^{n-1}b \end{bmatrix} = I$$

$$T^{-1}b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$T^{-1}AT = \begin{bmatrix} T^{-1}Ab & T^{-1}A^2b & \cdots & T^{-1}A^nb \end{bmatrix}$$

$$= \begin{bmatrix} 0 & & 0 & Y_1 \\ 1 & & & Y_2 \\ & & 0 & \vdots \\ 0 & & 1 & Y_n \end{bmatrix}, Y = T^{-1}A^nb$$

$$\text{suppose } \det\left(\lambda I - A\right) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0, \text{ C-H Thm} \Rightarrow A^n + a_{n-1}A^{n-1} + \cdots + a_0 I = 0$$

$$Y = T^{-1}A^nb = -a_{n-1}T^{-1}A^{n-1}b - \cdots - a_0T^{-1}b = \begin{bmatrix} -a_0 \\ -a_1 \\ \vdots \\ -a_{n-1} \end{bmatrix}$$

Drexel UNIVERSITY

Observability Form

$$\dot{x} = Ax + bu,\ y = cx$$

$$\mathcal{O} = \begin{bmatrix} c \\ cA \\ \vdots \\ cA^{n-1} \end{bmatrix},\ \det \mathcal{O} \neq 0$$

$$T = \mathcal{O} \Rightarrow$$

$$\dot{z} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_{n-1} & \cdots & -a_1 & -a_0 \end{bmatrix} z + \left(T^{-1}b\right)u,\ y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z$$

Observability Form – the transformation

$$y = cx$$

$$\dot{y} = cAx,\ cb = 0$$

$$\ddot{y} = cA^2x,\ cAb = 0$$

$$\vdots$$

$$y^{(n-1)} = cA^{n-1}x,\ cA^{n-2}b = 0$$

$$y^{(n)} = cA^n x + u,\ cA^{n-1}b = 1$$

$$\begin{aligned} z_1 &= cx \\ z_2 &= cAx \\ &\vdots \\ z_n &= cA^{n-1}x \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= cA^n S^{-1} z + u \end{aligned}$$

$$z = Sx$$

Observability Form – the transformation, cont'd

$$S = \begin{bmatrix} c \\ cA \\ \vdots \\ cA^{n-1} \end{bmatrix}, SS^{-1} = I \Rightarrow \begin{bmatrix} cS^{-1} \\ cAS^{-1} \\ \vdots \\ cA^{n-1}S^{-1} \end{bmatrix} = I$$

$$A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 I = 0$$

$$cA^n S^{-1} = -a_{n-1}cA^{n-1}S^{-1} - \cdots - a_1 cAS^{-1} - a_0 cS^{-1}$$

$$= \begin{bmatrix} -a_{n-1} & \cdots & -a_1 & -a_0 \end{bmatrix}$$

Summary of Forms

Observability

T_1

$$\dot{z} = \begin{bmatrix} 0 & & 0 & -a_0 \\ 1 & \ddots & & -a_1 \\ & \ddots & 0 & \vdots \\ 0 & & 1 & -a_{n-1} \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$y = \bar{c} z$$

controllability

T_2

$$\dot{z} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_{n-1} & \cdots & -a_1 & -a_0 \end{bmatrix} z + \bar{b} u$$

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z$$

$$\dot{z} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \bar{c} z$$

T_4

Controller/phase variable

$$\dot{z} = \begin{bmatrix} -a_{n-1} & 1 & & 0 \\ \vdots & 0 & \ddots & \\ -a_1 & & \ddots & 1 \\ -a_0 & 0 & & 0 \end{bmatrix} z + \bar{b} u$$

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z$$

T_3

Observer

Pole Placement Problem

Given a linear system:

$$\dot{x} = Ax + Bu$$

find a state feedback control:

$$u = Kx$$

such that the closed loop system:

$$\dot{x} = Ax + BKx = \left(A + BK\right)x$$

has a specified (self-conjugate) set of poles $\left\{p_1, p_2, \ldots, p_n\right\}$.

Pole Placement Sol'n: SISO Case

- Convert $\dot{x} = Ax + bu$ to controller form (phase variable form) using $x = Tz$:

$$\dot{z} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

- Set $u = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} z$ and obtain closed loop: $\dot{z} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ k_1 - a_0 & k_2 - a_1 & \cdots & k_n - a_{n-1} \end{bmatrix} z$

- Expand desired closed loop characteristic polynomial and compare coefficients, and solve for $k_1, \ldots, k_n$:

$$\phi_{cl}(\lambda) = (\lambda - p_1)(\lambda - p_2)\cdots(\lambda - p_n) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \cdots + \alpha_0 \Rightarrow \alpha_0 = a_0 - k_1, \alpha_1 = a_1 - k_2, \ldots, \alpha_{n-1} = a_{n-1} - k_n$$

- Convert back to x-coordinates: $Kz = KT^{-1}x \Rightarrow u = \left(KT^{-1}\right)x$

Drexel UNIVERSITY

Pole Place Design: The Easy Way

PLACE Pole placement technique

K = PLACE(A,B,P) computes a state-feedback matrix K such that the eigenvalues of A-B*K are those specified in vector P. No eigenvalue should have a multiplicity greater than the number of inputs.

Warning!! Notice the sign difference.

Ackermann's Formula

$$K = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \mathcal{C}^{-1} \phi_{cl}(A) \qquad L = \phi_{cl}(A) \mathcal{O}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

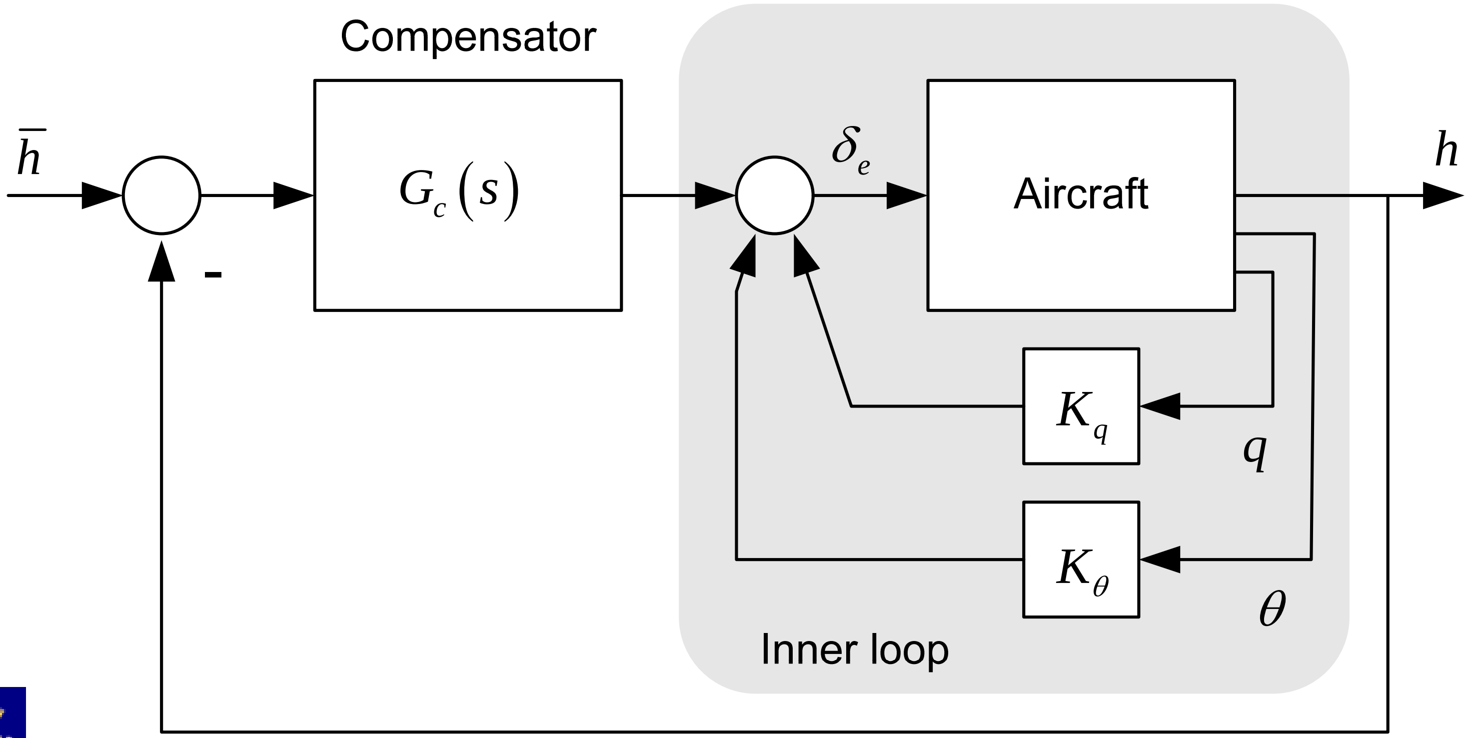
ACKER Pole placement gain selection using Ackermann's formula.

K = ACKER(A,B,P) calculates the feedback gain matrix K such that the single input system

```
.
x = Ax + Bu
```

with a feedback law of u = -Kx has closed loop poles at the values specified in vector P, i.e., P = eig(A-B*K).

Boeing 747-400
altitude hold controller



Boeing 747 Dynamics (cruise)

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} -0.006 & 0.0263 & 0 & -32.2 & 0 \\ -0.0941 & -0.624 & 820 & 0 & 0 \\ -0.000222 & -0.00153 & -0.668 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 830 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} 0 \\ -32.7 \\ -2.08 \\ 0 \\ 0 \end{bmatrix} \delta_e$$

$$h = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix}$$

Drexel UNIVERSITY

Boeing 747 Inner Loop Design

```
A=[-0.0064 0.0263 0 -32.2 0;-0.0941 -0.624 820 0 0;-
.000222 -0.00153 -0.668 0 0;0 0 1 0 0;0 -1 0 830 0];
B=[0;-32.7;-2.08;0;0];
C=[0 0 0 0 1];
poles=[0,-2.25+2.99i,-2.25-2.99i,-0.0105,-0.0531];
Kinner=place(A,B,poles)
eig(A-B*Kinner)
Kinner =
   -0.0008   -0.0054   -1.4845   -0.6517         0
ans =
        0
  -2.2500 + 2.9900i
  -2.2500 - 2.9900i
  -0.0531
  -0.0105
```

Small contribution, so we'll drop these two terms

Boeing 747 cont'd

$$\delta_e \to h : G(s) = \frac{32.7(s+0.0045)(s+0.5645)(s-5.61)}{s\underbrace{(s+0.003\pm j0.0098)}_{\text{phugoid}}\underbrace{(s+0.6463\pm j1.1211)}_{\text{short-period}}}$$

Choose: $K_q = -1.4845,\ K_\theta = -0.6517$

Inner loop improves stability

$$A \to A_p = A + b\begin{bmatrix} 0 & 0 & -0.8 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0064 & 0.0263 & 0 & -32.2 & 0 \\ -0.0941 & -0.624 & 721 & -21 & 0 \\ -0.0002 & -0.0015 & -3.76 & -1.36 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 830 & 0 \end{bmatrix}$$

$$G \to G_p(s) = \frac{32.7(s+0.0045)(s+0.5645)(s-5.61)}{s(s+0.0105)(s+0.0531)\underbrace{(s+2.25\pm j2.99)}_{\text{short-period}}}$$

Boeing 747 cont'd

- Inner loop 'stabilizer' requires only the allowed measurements, q, θ – so it can be readily implemented
- The outer loop - 'altitude hold' – only uses altitude. As we will see, it is necessary to estimate other states

Computations - Attitude hold state feedback gain

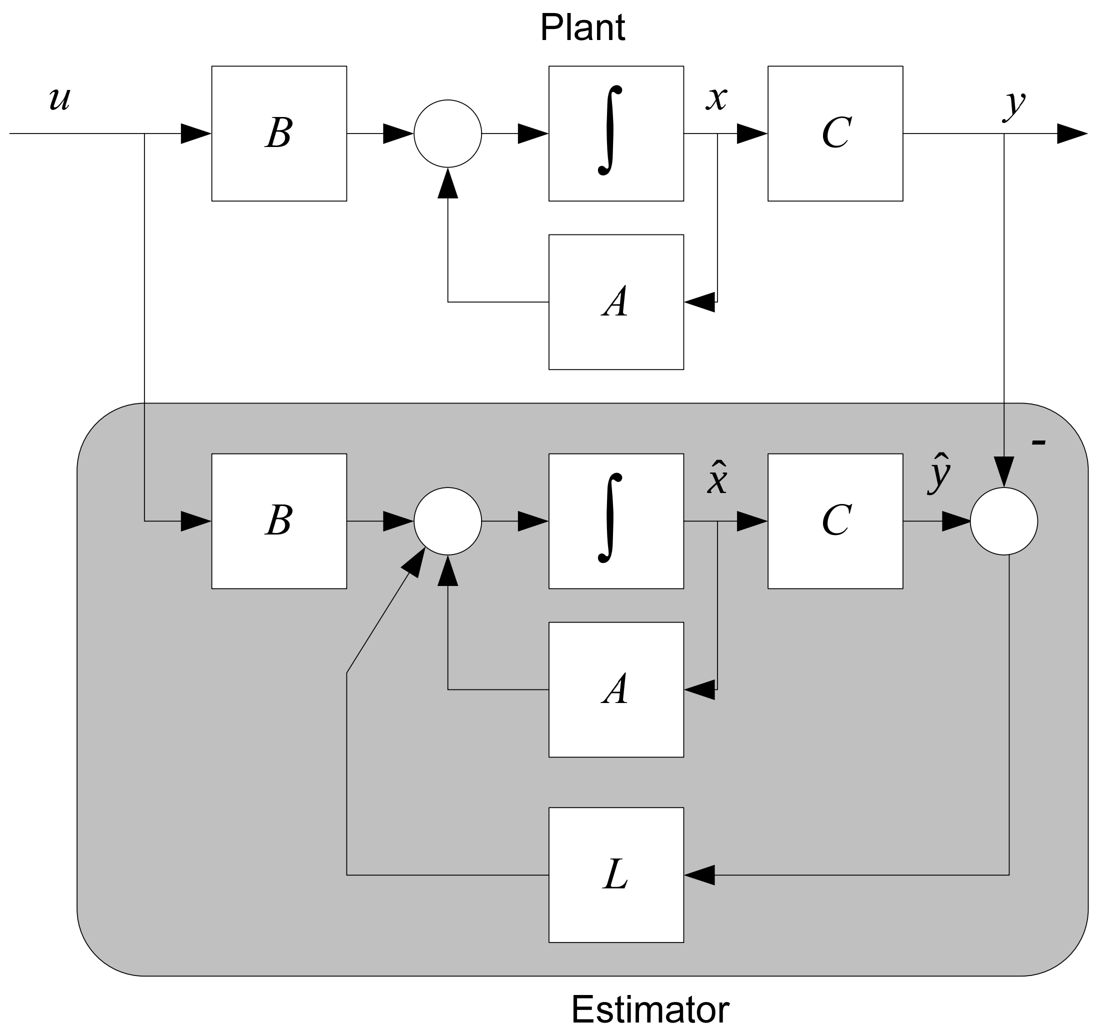
```
>> A=[-0.0064 0.0263 0 -32.2 0;-0.0941 -0.624 761 -196.2 0;
                       -.0002 -0.0015 -4.41 -12.48 0;0 0 1 0 0;0 -1 0 830 0]
A =
   -0.0064    0.0263         0  -32.2000         0
   -0.0941   -0.6240  761.0000 -196.2000         0
   -0.0002   -0.0015   -4.4100  -12.4800         0
         0         0    1.0000         0         0
         0   -1.0000         0  830.0000         0
>> b=[0;-32.7;-2.08;0;0]
b =
         0
  -32.7000
   -2.0800
         0
         0
>> p=[-.0045;-.145;-.513;-2.25+i*2.98;-2.25-i*2.98]
  -0.0045
  -0.1450
  -0.5130
  -2.2500 + 2.9800i
  -2.2500 - 2.9800i
```

Computations - Attitude hold state feedback gain

```
>> K=place(A,b,p)
K =
   -0.0011    0.0016   -0.0843   -1.6011   -0.0010
>> eig(A-b*K)
ans =
  -2.2500 + 2.9800i
  -2.2500 - 2.9800i
  -0.0045
  -0.5130
  -0.1450
```

Clearly, of the five states u,w,q,θ,h, the weights on θ, and then q, are the largest. The other 3 are of the same order. We will estimate all 5 states from measurement h.

Full-State Estimator



Estimator Error Dynamics

$$\dot{x} = Ax + Bu,\ y = Cx$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y),\ \hat{y} = C\hat{x}$$

$$e := x - \hat{x} \Rightarrow \dot{e} = Ae + LCe \Rightarrow \boxed{\dot{e} = (A + LC)e}$$

One approach is to select L so as to place the poles of $(A + LC)$. Notice that the following two pole placement problems are equivalent:

$(A + BK), (A, B)$ controllable

$(A^T + C^T L^T), (A, C)$ observable

Closed Loop Dynamics

$$\dot{x} = Ax + Bu$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L\left(C\hat{x} - Cx\right)$$

$$u = K\hat{x}$$

$$\dot{x} = Ax + BK\hat{x} = \left(A + BK\right)x - BKe$$

$$\dot{e} = Ae + LCe = \left(A + LC\right)e$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ 0 & A + LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \Rightarrow \lambda\left(A + BK\right) + \lambda\left(A + LC\right)$$

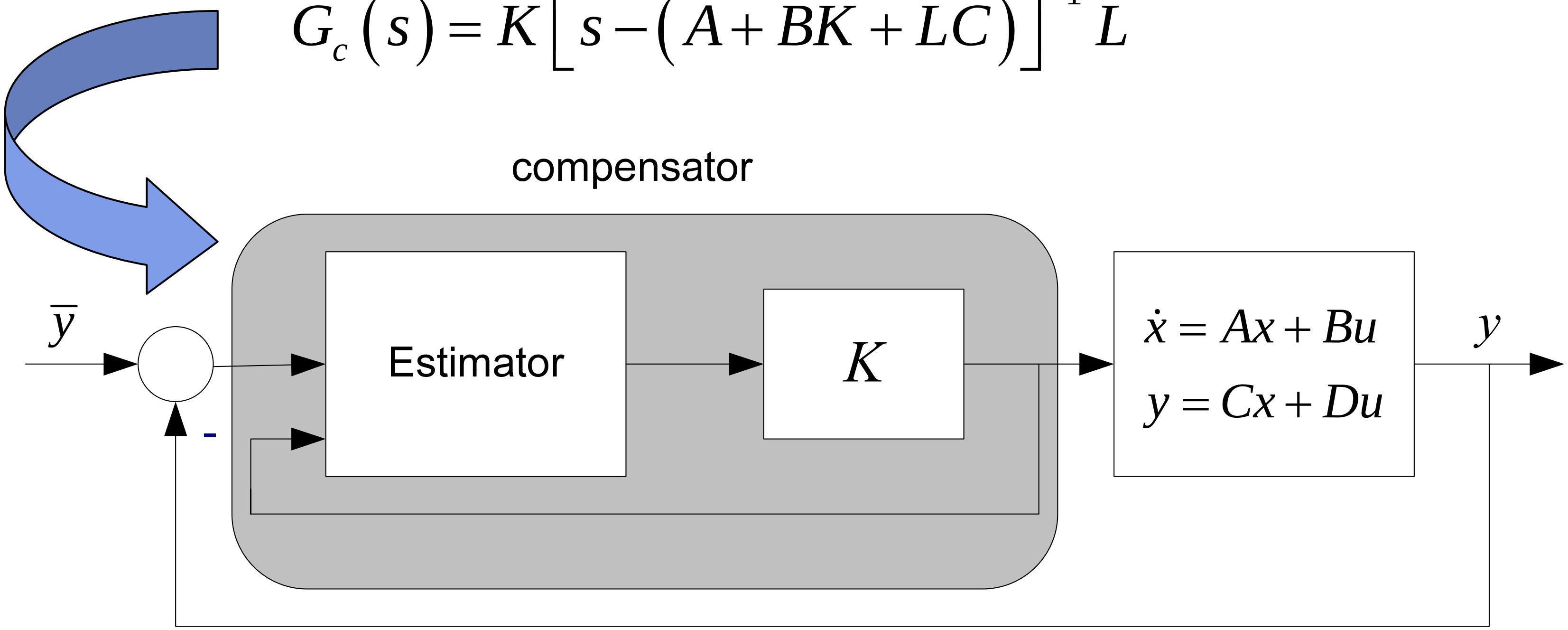
closed loop poles

Compensator

$$\dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y) \Rightarrow (A + BK + LC)x + Le$$

$$u = K\hat{x}$$

$$G_c(s) = K\left[s - (A + BK + LC)\right]^{-1} L$$

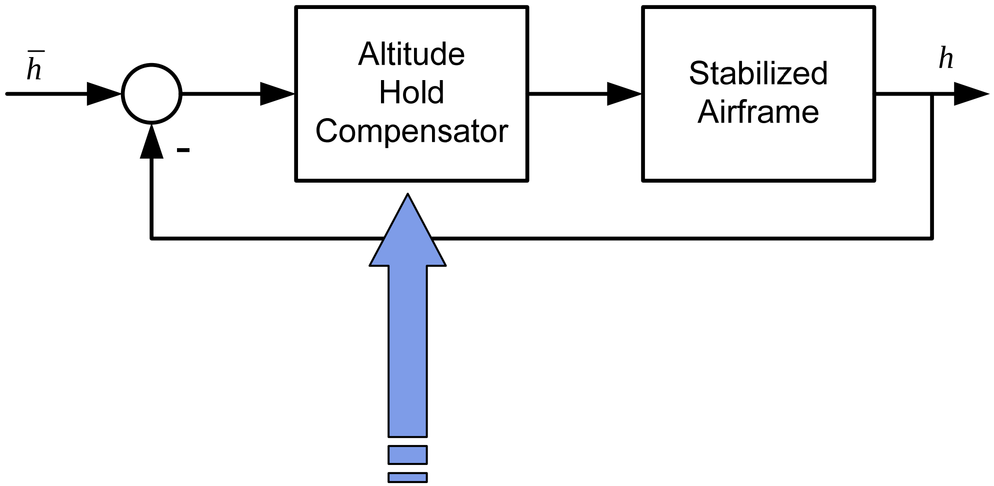


Computations – observer gain

```
>> c=[0,0,0,0,1];
>> poles=[-0.0045,-5.645,-9,-10,-11];
>> L=place(A',c',poles)'
L =
   -6.5323
  915.2339
   -2.7283
    1.4615
   30.6091
>> eig(A-L*c)
ans =
   -0.0045
  -11.0000
  -10.0000
   -9.0000
   -5.6450
```

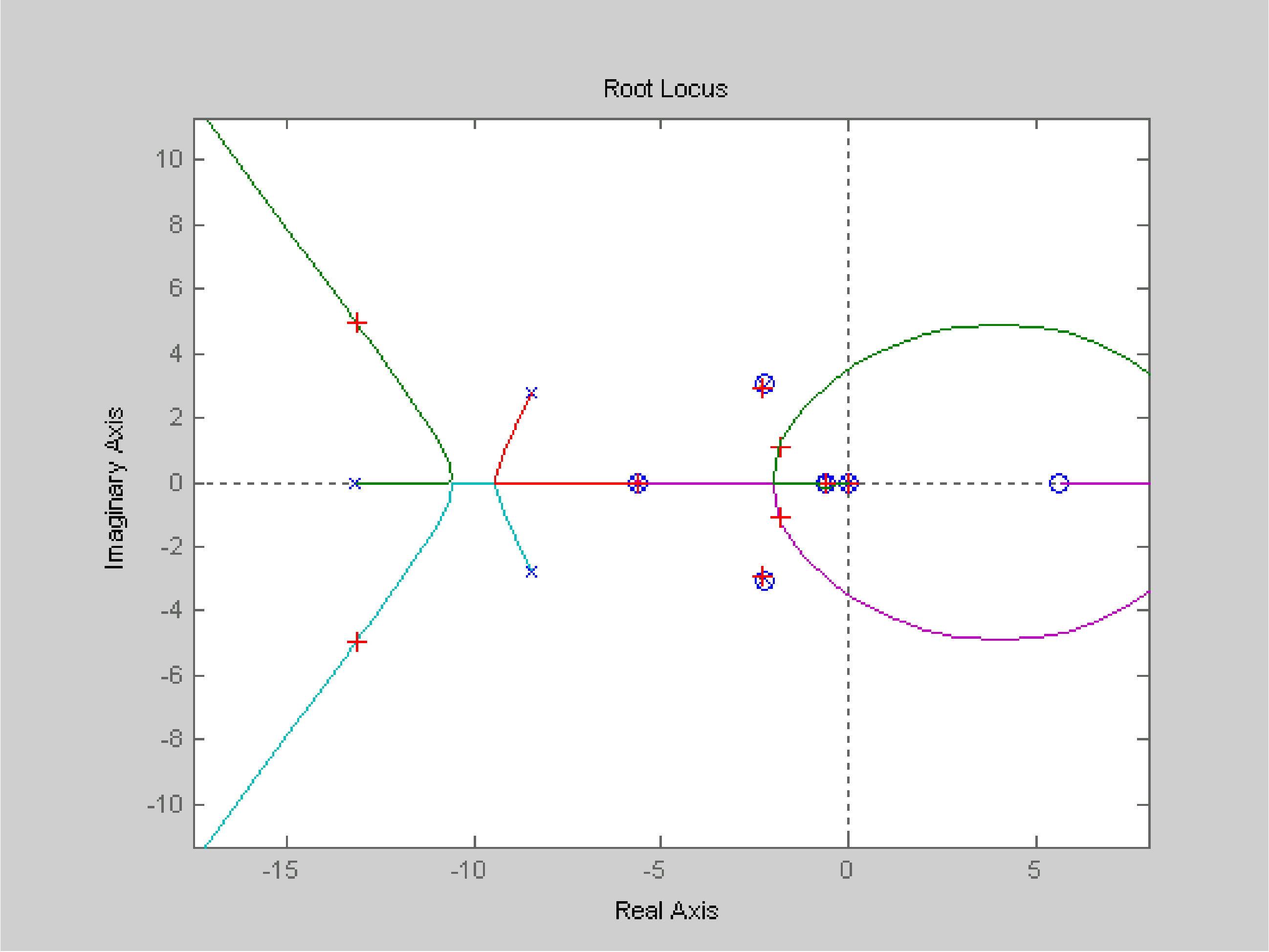
Computations – compensator transfer function

We have the plant transfer function, now we will compute the compensator transfer function. This will allow us to compute root locus and margins.

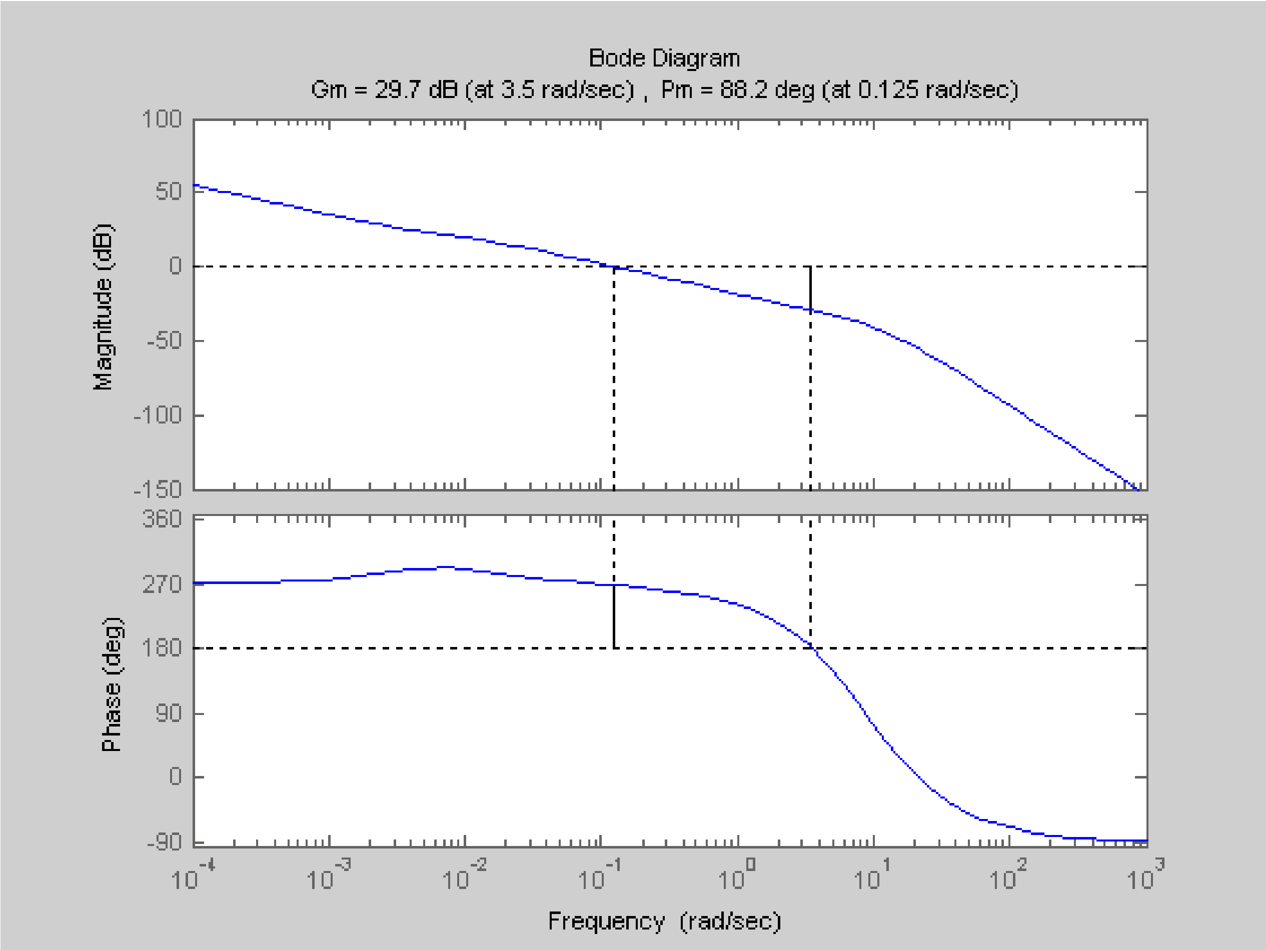



```
>> Ac=A-b*K-L*c;
>> Bc=L;
>> Cc=K;
>> Gcss=ss(Ac,Bc,Cc,0);
>> Gc=tf(Gcss);
>> zpk(Gc)
Zero/pole/gain:
-0.64364 (s+0.5803) (s+0.004708) (s^2  + 4.517s + 14.29)
--------------------------------------------------------
 (s+13.22) (s+5.644) (s+0.004507) (s^2  + 16.9s + 79.2)
```

Root Locus

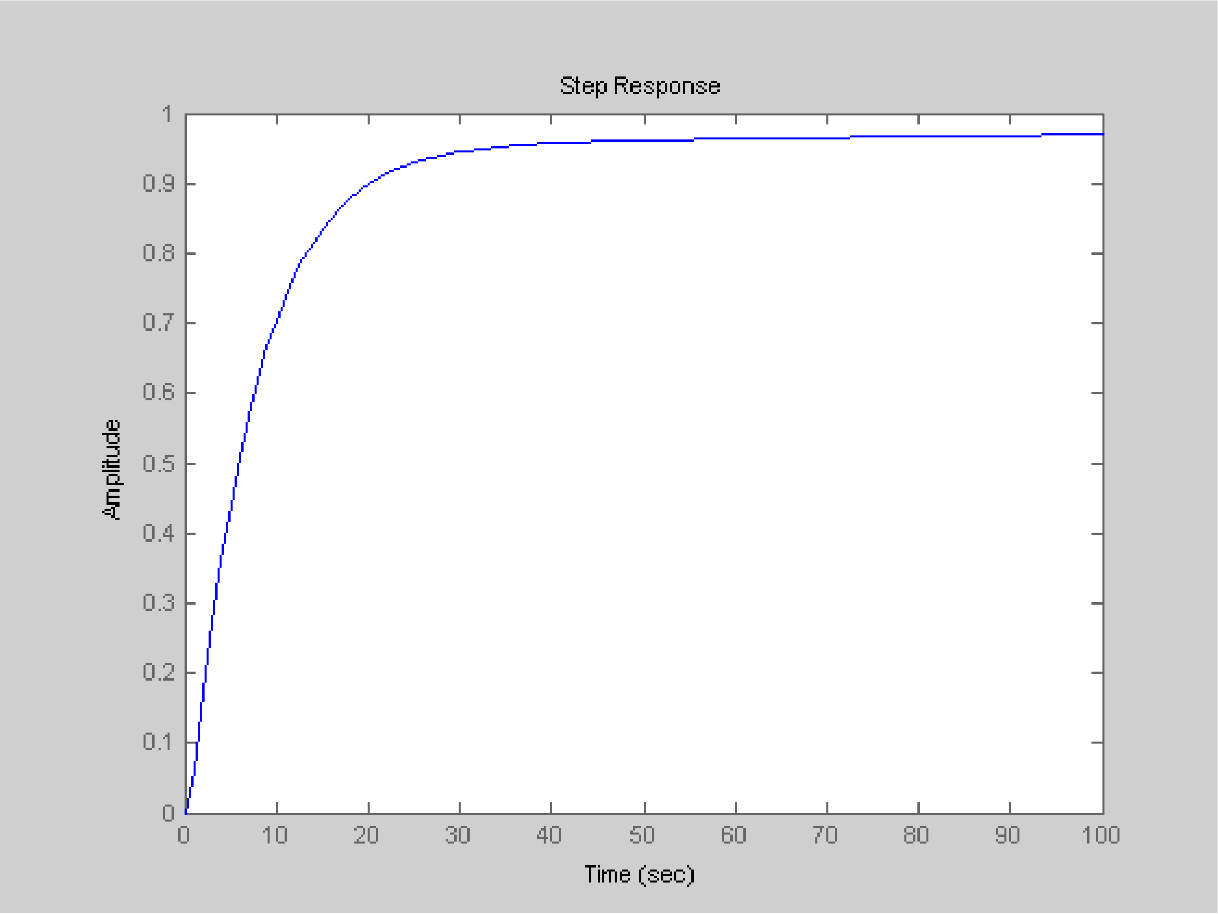

Root Locus
10
8
6
4
2
0
-2
-4
-6
-8
-10
Imaginary Axis
-15
-10
-5
0
5
Real Axis

Margins

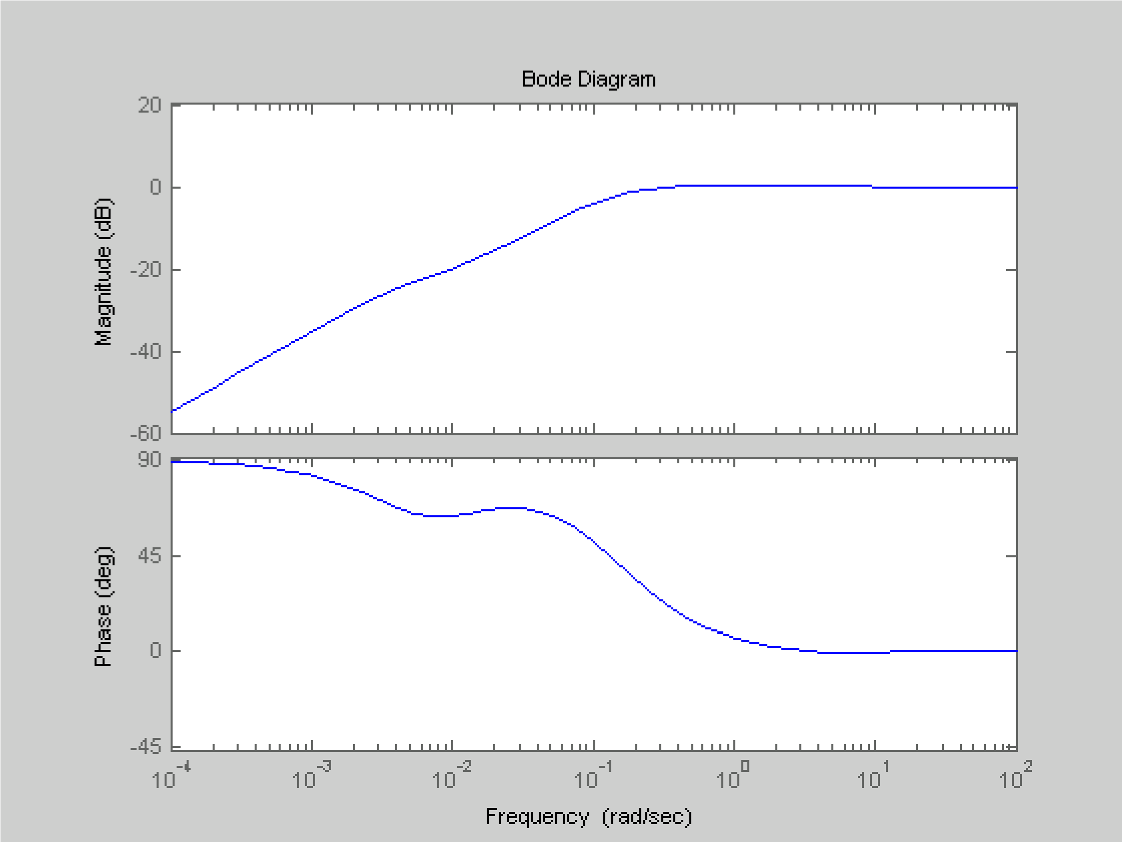


Bode Diagram
Gm = 29.7 dB (at 3.5 rad/sec) , Pm = 88.2 deg (at 0.125 rad/sec)
Magnitude (dB)
100
50
0
-50
-100
-150
Phase (deg)
360
270
180
90
0
-90
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

Step response



Sensitivity Function



Example: F-16

landing approach

$$\begin{bmatrix}\dot{u}\\ \dot{\alpha}\\ \dot{q}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}-0.507 & -3.861 & 0 & -32.17\\ -0.00117 & -0.5164 & 1 & 0\\ -0.000129 & 1.4168 & -0.4932 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}u\\ \alpha\\ q\\ \theta\end{bmatrix}+\begin{bmatrix}0\\ -0.0717\\ -1.645\\ 0\end{bmatrix}\delta_E$$

$$y=\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}u\\ \alpha\\ q\\ \theta\end{bmatrix}$$

$$\text{phugoid: } \lambda=-0.0438167\pm j0.206461 \quad h=\begin{bmatrix}0.999978\\ 0.000484\\ 0.001343\\ -0.000272\end{bmatrix}\pm j\begin{bmatrix}0\\ 0.0002676\\ 0.0002264\\ -0.0064497\end{bmatrix}$$

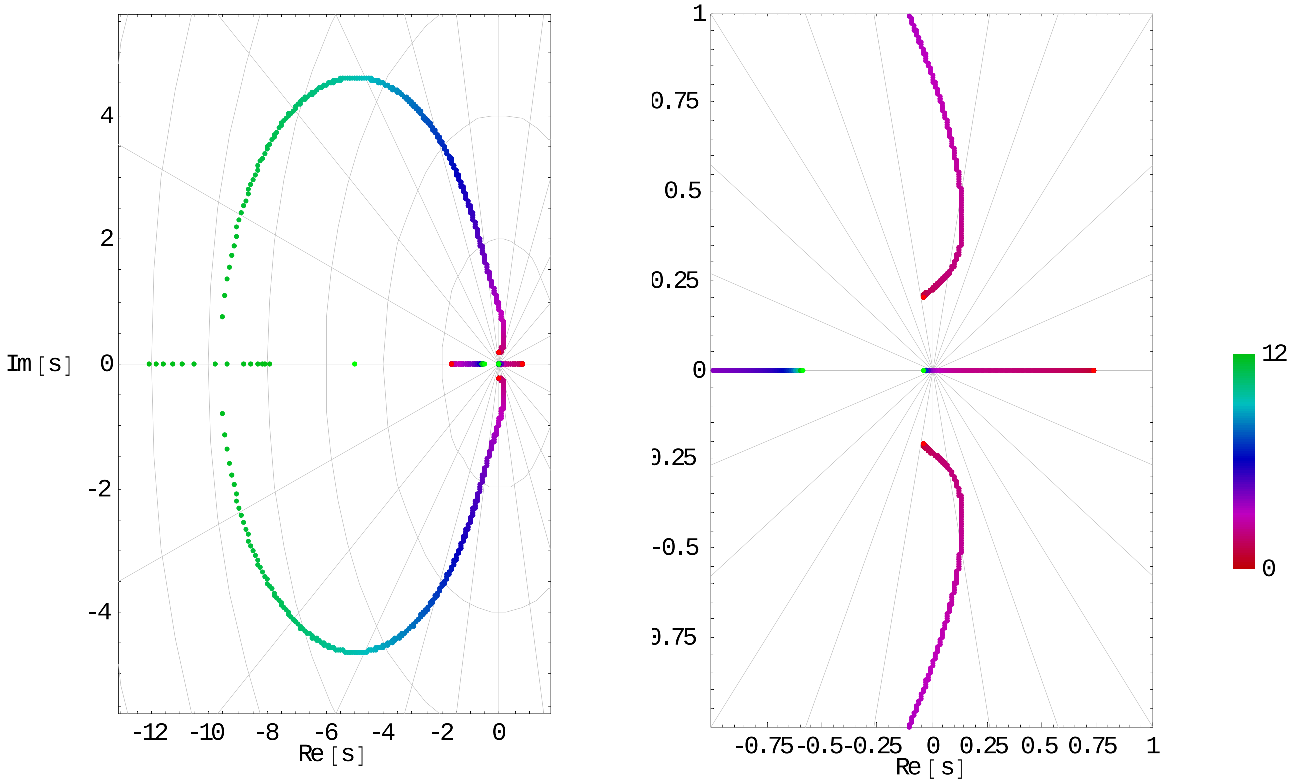
$$\text{short period: } \lambda=-1.7036,\ 0.730937 \quad h=\begin{bmatrix}-0.994287\\ -0.063373\\ 0.074073\\ -0.043481\end{bmatrix},\begin{bmatrix}0.999508\\ -0.014171\\ -0.016507\\ -0.022584\end{bmatrix}$$

Drexel UNIVERSITY

F-16: PI Control

$$G_p(s) = 1.645\frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = \frac{s+5}{s}$$



Example: F-16 state feedback

Desired poles -

short period: $\lambda_{1,2} = -1.25 \pm j2.16506$

phugoid: $\lambda_{3,4} = -0.01 \pm j0.0994987$

$$K = \begin{bmatrix} 0.004076 & 3.87578 & 0.718424 & 0.095189 \end{bmatrix}$$

Example: F-16 state feedback

Desired poles -

short period: $\lambda_{1,2} = -1.25 \pm j2.16506$

phugoid: $\lambda_{3,4} = -0.01 \pm j0.0994987$

$$\operatorname{Re} R_{\lambda_1} = \begin{bmatrix} -0.942209 \\ -0.049274 \\ -0.088225 \\ -0.029211 \\ \hline 0.260982 \end{bmatrix}, \quad \operatorname{Im} R_{\lambda_1} = \begin{bmatrix} 0 \\ -0.048293 \\ 0.135264 \\ -0.057615 \\ \hline 0.095482 \end{bmatrix}$$

$$\operatorname{Re} R_{\lambda_3} = \begin{bmatrix} -0.999992 \\ 0.001453 \\ -0.000320 \\ 0.001090 \\ \hline -0.001428 \end{bmatrix}, \quad \operatorname{Im} R_{\lambda_3} = \begin{bmatrix} 0 \\ 0.000117 \\ -0.000077 \\ -0.003107 \\ \hline -0.000104 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.004076 & 3.87578 & 0.718424 & 0.095189 \end{bmatrix}$$

Example: F-16 Rynaski “robust observer”

"place observer poles at LHP plant zeros, remainder are placed arbitrarily"

$$\lambda = 0, -0.04231, -0.5865, -1$$

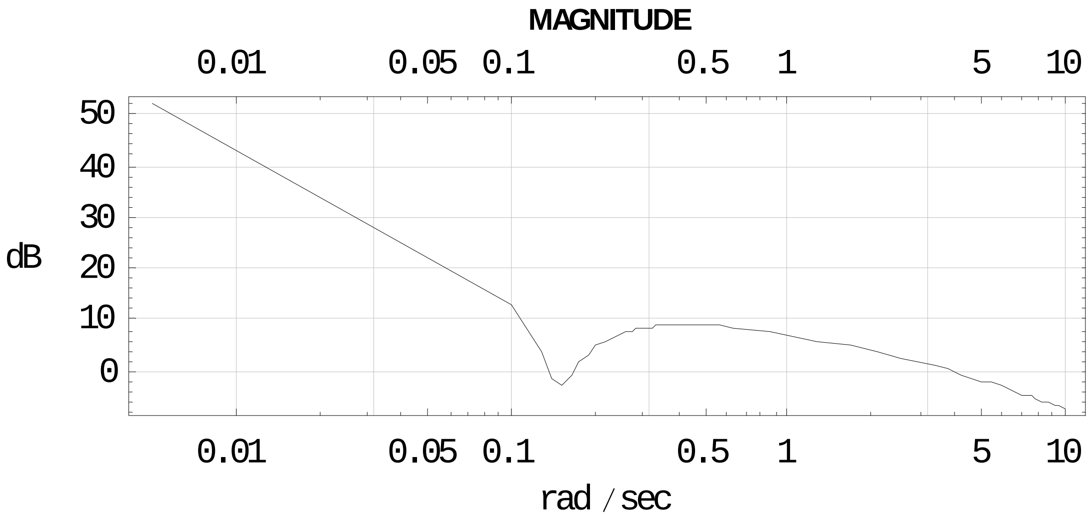
$$R_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{0.707107}{0.707107} \end{bmatrix}, R_2 = \begin{bmatrix} 0.000934 \\ -0.006733 \\ 0.000293 \\ \dfrac{0.710515}{0.703649} \end{bmatrix}, R_3 = \begin{bmatrix} 0.001445 \\ 0.665243 \\ -0.028975 \\ \dfrac{0.079296}{0.741837} \end{bmatrix}, R_4 = \begin{bmatrix} 0.000863 \\ 0.73183 \\ -0.247431 \\ \dfrac{0.027934}{0.634367} \end{bmatrix}$$

$$L^T = \begin{bmatrix} 0.168343 & -1.02106 & -0.56851 & -1 \end{bmatrix}$$

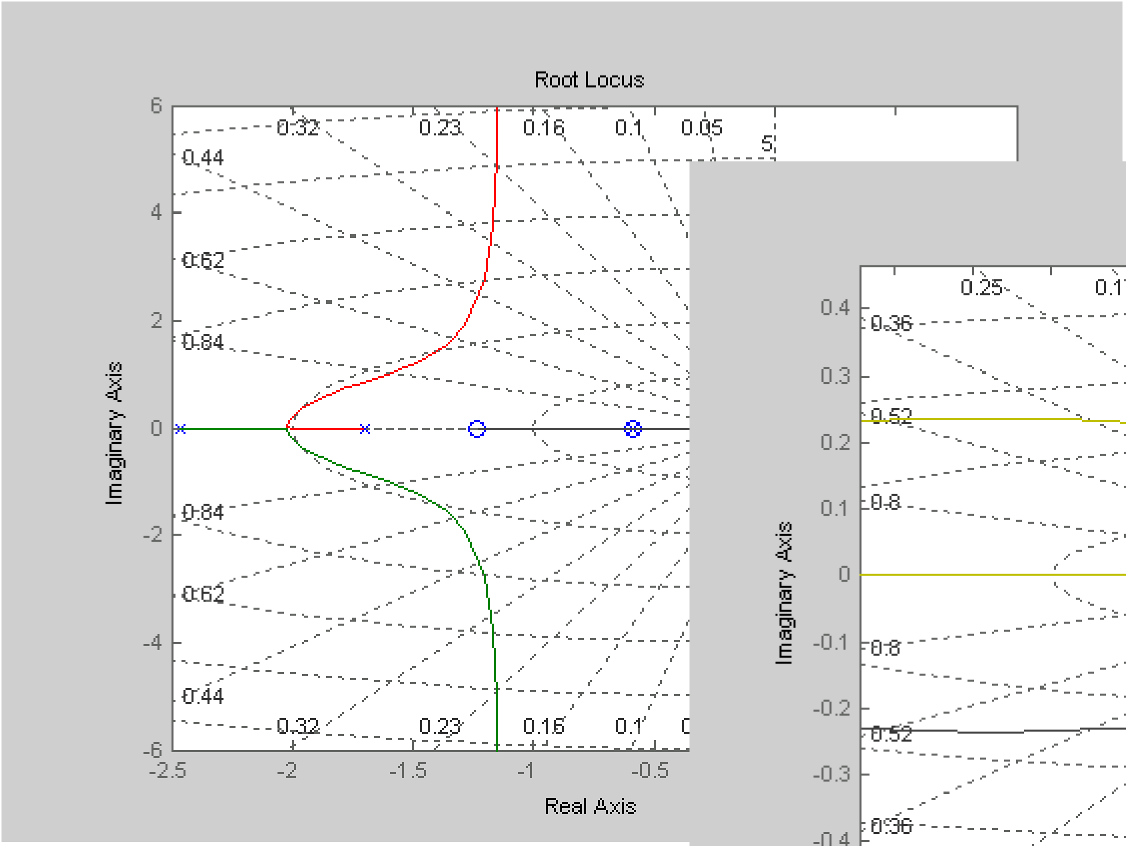
Example: F-16

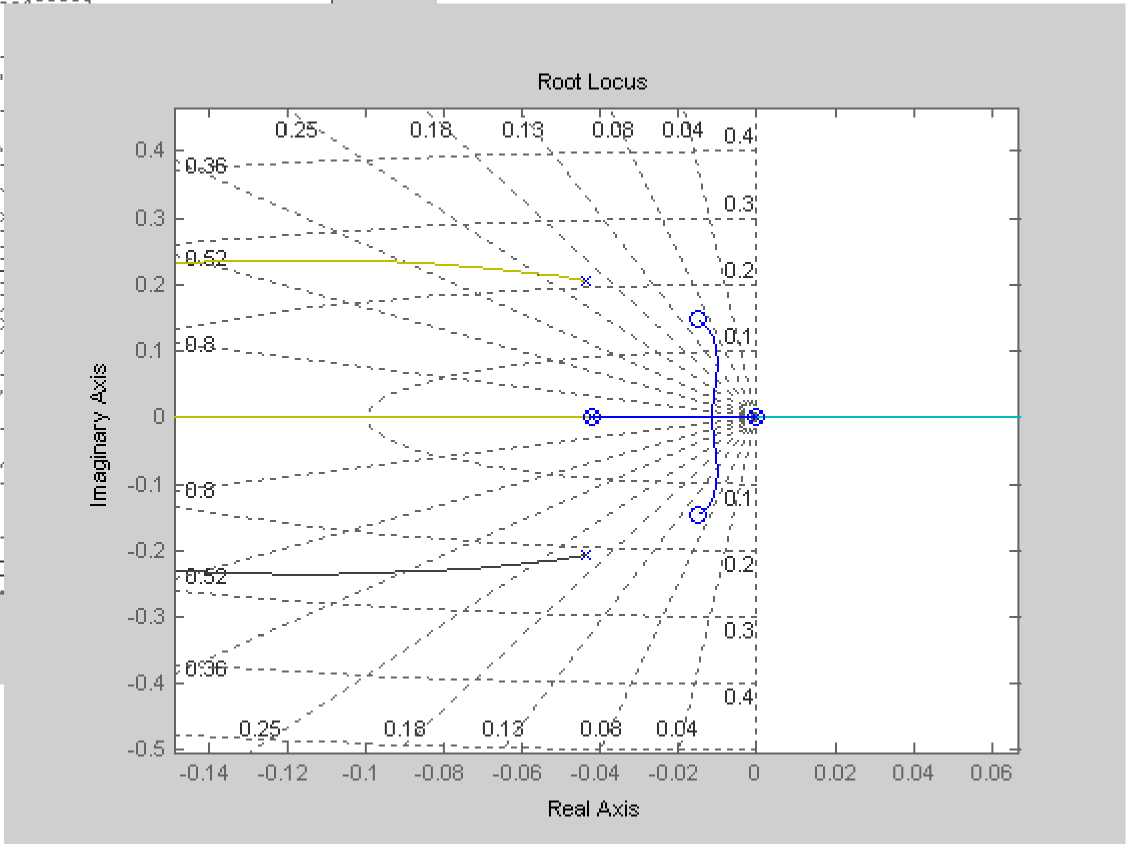
$$G_p(s) = 1.645\frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = 4.46035\frac{(s+2.45962)(s+0.0148335\pm j0.147508)}{s(s+0.423102)(s+0.586577)(s+2.45962)}$$

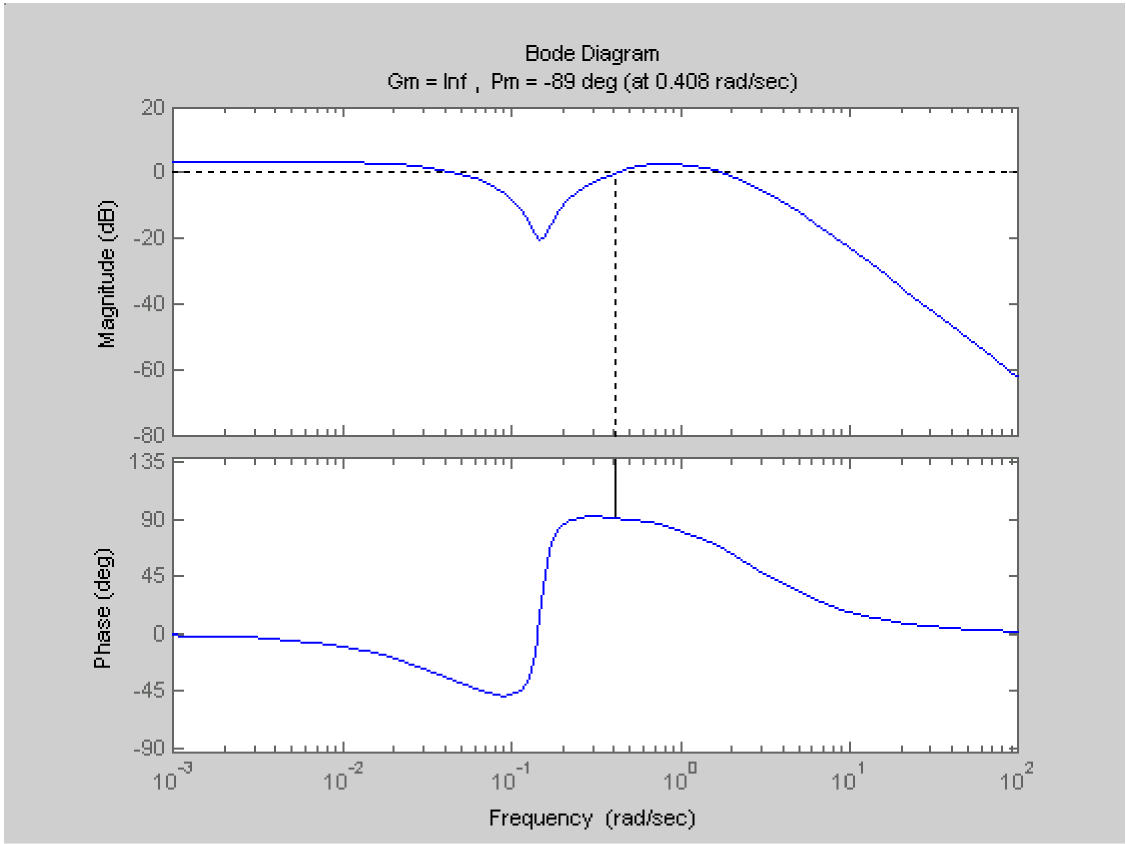


Root Locus


Root Locus
Imaginary Axis
Real Axis

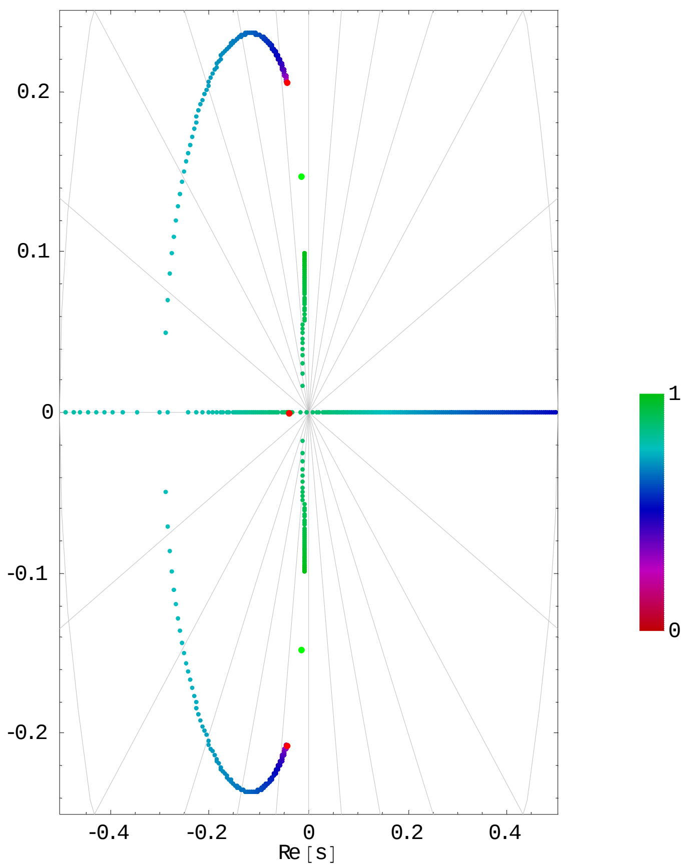

Root Locus
Imaginary Axis
Real Axis

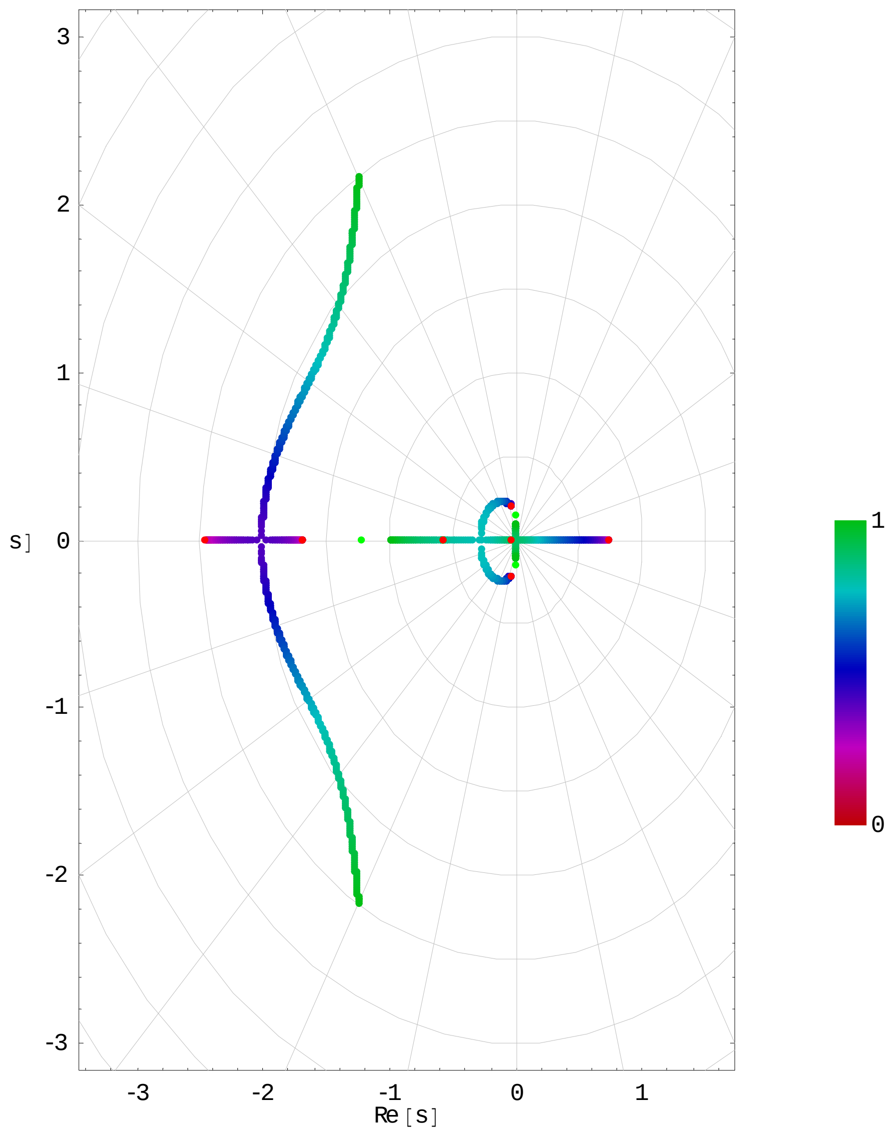
Margins



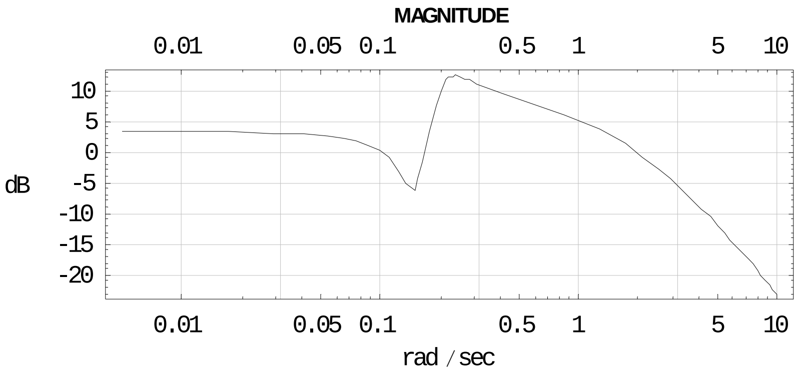
Bode Diagram
Gm = Inf , Pm = -89 deg (at 0.408 rad/sec)
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)

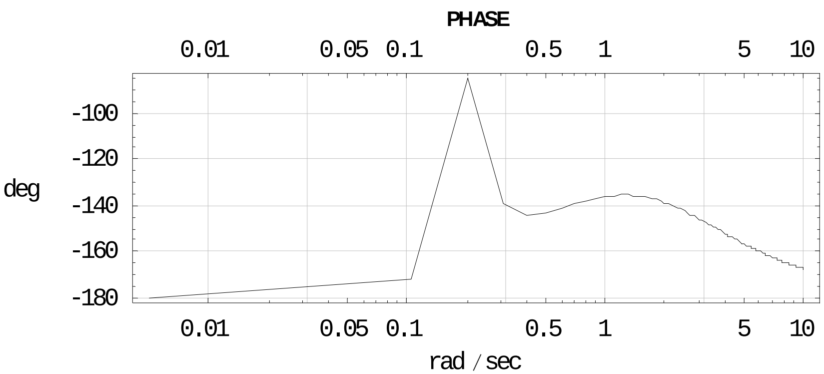
Example F-16

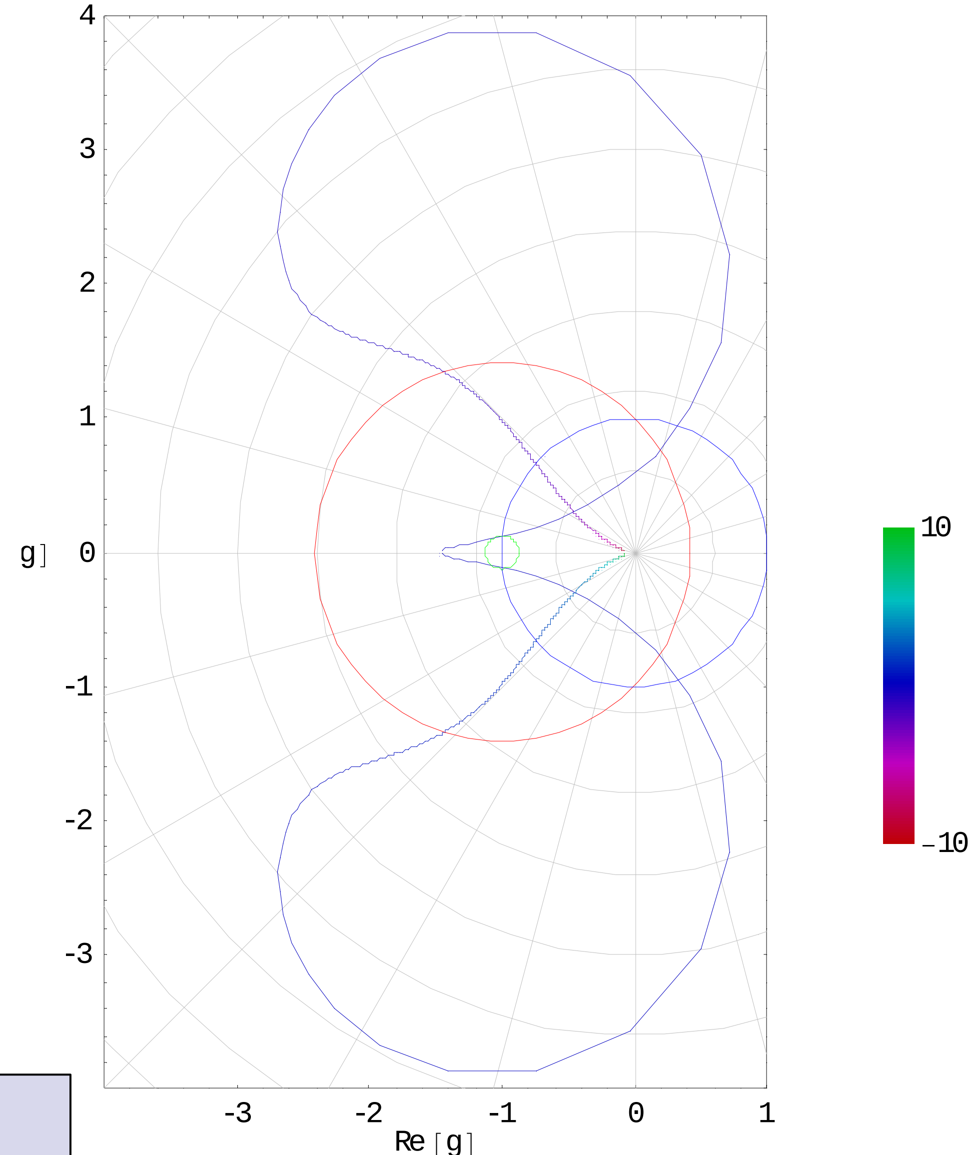




Example: F-16







Sensitivity Function